Intrinsic trapping of stochastic sheared magnetic field lines

M. Negrea and I. Petrisor
Department of Physics, University of Craiova, A.I. Cuza str. 13, Craiova, Dolj 1100, Romania Association Euratom-MEC, Romania

R. Balescu
Department of Statistical Physics and Plasma, Association Euratom-Etat Belge-sur la Fusion, CP 231, Universite Libre de Bruxelles, Campus Plaine, 1050 Bruxelles, Belgium

(Received 30 September 2003; revised manuscript received 12 July 2004; published 14 October 2004)

The decorrelation trajectory method is applied to the diffusion of magnetic field lines in a perturbed sheared slab magnetic configuration. Some interesting decorrelation trajectories for several values of the magnetic Kubo number and of the shear parameter are exhibited. The asymmetry of the decorrelation trajectories appears in comparison with those obtained in the purely electrostatic case studied in earlier work. The running and asymptotic diffusion tensor components are calculated and displayed.

DOI: 10.1103/PhysRevE.70.046409 PACS number(s): 52.35.Ra, 52.25.Fi, 05.40.−a, 02.50.−r

I. INTRODUCTION

A central issue for fusion is the description of turbulence phenomena in a plasma at high temperature. The magnetic turbulence appears as a plausible candidate for determining the anomalous transport properties of a hot magnetized plasma. The magnetic fluctuations, whose intensity is measured by the dimensionless magnetic Kubo number K_m (to be defined below), even when small, can destroy the nested magnetic configurations in a toroidal confinement geometry, such as in a tokamak. Moreover, in such disturbed regions where the magnetic field is completely stochastic, a single magnetic line fills a three-dimensional region. As a result, any particle makes radial displacements, thus enhancing the radial transport. The study of transport in such stochastic magnetic fields is therefore of great importance for the fusion.

In the present paper we restrict ourselves to the study of the geometrical aspect of the problem: we thus analyze the properties of the magnetic lines of a sheared stochastic magnetic field alone. We consider the simplified model of the slab approximation including the shear. The presence of the shear influences the length of a magnetic line between two fixed z values (z is a coordinate in the direction of the reference magnetic field). It will be shown in the following that the magnetic shear and the magnetic Kubo number K_m have an important influence on the diffusion of the magnetic field lines. The problem is simplified whenever the study is restricted to stochastic magnetic fields with small amplitudes and/or large perpendicular correlation lengths for which the magnetic Kubo number K_m is small. The main contribution of our work consists, however, in analyzing the influence of the shear in both weak- and strong-turbulence regimes for the stochastic magnetic fields. Until now the influence of the shear on the diffusion coefficients was analyzed only in the quasilinear limit—i.e., the small-Kubo-number regime. The diffusion coefficients for the magnetic field lines were studied for both small- and large-Kubo-number regimes in many papers (see, e.g., [1,2]) but for the shearless case.

Our analysis is not restricted only in a range of small magnetic Kubo number $K_m < 1$. Using a recently developed method of investigation of the diffusion in a stochastic velocity field [3] we extend our analysis to a relatively large-Kubo-number regime $K_m > 1$. Because of the existence of the two parameters (to be defined below)—the magnetic Kubo number K_m and the shear parameter θ_s—a richer class of behaviors of the diffusion coefficients is observed. The competition between these two parameters plays an important role and seems to be decisive in the determination of the trapping effects.

The paper is organized as follows. The Langevin equations for the sheared stochastic magnetic field are established in Sec. II. In Sec. III, in the framework of decorrelation trajectory (DCT) method, the expressions of the Lagrangian correlations and of the diffusion tensor are obtained. In Sec. IV the decorrelation trajectories are analyzed. In Sec. V the running and asymptotic diffusion coefficients are studied. The conclusions are summarized in Sec. VI.

II. LANGEVIN EQUATIONS FOR THE MAGNETIC FIELD LINES

In the present paper we consider a magnetic field of the following form [4]:

$$\mathbf{B}(X,Y,Z) = B_0\{\mathbf{e}_z + \beta b_x(X,Y,Z)\mathbf{e}_x + [\beta b_y(X,Y,Z) + L_s^{-1}X]\mathbf{e}_y\}, \quad (1)$$

where β is a dimensionless parameter measuring the amplitude of the magnetic field fluctuations relative to the main constant magnetic field B_0.

The linear term depending on X in the right-hand side of Eq. (1) is the shear term, L_s is the shear length, and Eq. (1) is the so-called "sheared slab" configuration which must be understood as a local approximation. This approximation mimicks the field around a rational surface $X=0$ existing in toroidal systems and is valid only for $|X| \ll L_s$. The magnetic field fluctuations are described by the dimensionless functions $b_i(X,Y,Z), i=(x,y)$, which depend on the spatial coordinates X,Y,Z but may also depend on time t. We will study in this paper only the stationary case. The fluctuation

$b_i(X,Y,Z)$ is taken to be a Gaussian random process which is fully specified by its binary correlation. If we consider the system with a quasi gyrotropical symmetry (because of existence of the shear)—i.e., quasi-isotropy in a plane perpendicular to the main magnetic field B_0— the two characteristic lengths in this plane are equal to each other: $\lambda_x=\lambda_y\equiv\lambda_\perp$. The third characteristic length is $\lambda_z\equiv\lambda_\parallel$.

We introduce the coordinates x,y,z which are dimensionless quantities related to the dimensional ones X,Y,Z by the relations

$$x=\frac{X}{\lambda_\perp},\quad y=\frac{Y}{\lambda_\perp},\quad z=\frac{Z}{\lambda_\parallel},\tag{2}$$

where $\lambda_\perp$ and $\lambda_\parallel$ are the characteristic lengths characterizing the turbulent state; they are called, respectively, perpendicular and parallel correlation lengths. An additional characteristic length is related to the shear: L_s. We thus construct two dimensionless parameters describing the stochastic magnetic geometry:

$$K_m=\beta\frac{\lambda_\parallel}{\lambda_\perp},\quad \theta_s=\frac{\lambda_\parallel}{L_s}.\tag{3}$$

The equations for the magnetic field lines are obtained from Eq. (1):

$$\frac{dX}{\beta B_0 b_x}=\frac{dY}{\beta B_0 b_y+B_0L_s^{-1}X}=\frac{dZ}{B_0}.\tag{4}$$

These equations can be rewritten by considering the Z variable as a mere parameter which plays the role of “time”:

$$\frac{dX}{dZ}=\beta b_x(X,Y;Z),$$

$$\frac{dY}{dZ}=\beta b_y(X,Y,Z)+L_s^{-1}X.\tag{5}$$

Using Eq. (2), Eq. (1) becomes

$$\mathbf{B}(x,y,z)=B_0\{\mathbf{e}_z+\beta b_x(x,y,z)\mathbf{e}_x+[\beta b_y(x,y,z)+\lambda_\perp L_s^{-1}x]\mathbf{e}_y\}.\tag{6}$$

The magnetic field given in Eq. (1) or Eq. (6) must satisfy the zero-divergence constraint $\nabla\cdot\mathbf{B}=0$ imposed by Maxwell's equations. This condition is automatically fulfilled if we consider that the fluctuating magnetic field derives from the following vector potential which has only a z component:

$$\mathbf{A}(X,Y,Z)=B_0\lambda_\perp\beta\ \psi(x,y,z)\mathbf{e}_z.\tag{7}$$

Equations (5) can then be rewritten in the following dimensionless form:

$$\frac{dx(z)}{dz}=K_m b_x[x(z),y(z),z]=K_m\left.\frac{\partial\psi(x,y,z)}{\partial y}\right|_{\mathbf{x}=\mathbf{x}(z)},$$

$$\frac{dy(z)}{dz}=K_m b_y[x(z),y(z),z]+\theta_s x(z)$$
$$=-K_m\left.\frac{\partial\psi(x,y,z)}{\partial x}\right|_{\mathbf{x}=\mathbf{x}(z)}+\theta_s x(z).\tag{8}$$

The system (5) is of the same form as the system for the guiding center motion of a test particle in a fluctuating electrostatic field, with the addition of the shear contribution [3]. It also represents the characteristic system of equations for a Liouville stochastic equation (or hybrid kinetic equation) for the density probability function; the shear term in Eqs. (5) is an advection term in the Liouville stochastic equation and is not a directly fluctuating term. We deal in Eqs. (5) with a single vectorial stochastic function with the components $b_i(x,y;z), i=(x,y)$. The fluctuating potential function $\psi(x,y;z)$ is assumed to be Gaussian, spatially homogeneous, gyrotropic, and with zero average. The second-order moment of ψ— i.e., the Eulerian autocorrelation function of the dimensionless potential—is assumed to be of the following factorized form:

$$E(x,y,z)=\langle\psi(0,0,0)\psi(x,y,z)\rangle=E_1(r)E_2(z),\tag{9}$$

where $r=\sqrt{x^2+y^2}$ and:

$$E_1(r)=\exp\left(-\frac{x^2+y^2}{2}\right),\quad E_2(z)=\exp\left(-\frac{z^2}{2}\right).\tag{10}$$

The Fourier transform of $E(x,y,z)$, is very useful in deriving the correlations between the stochastic functions and has the following form:

$$\tilde{E}(k,k_\parallel)\equiv\tilde{E}_1(k)\tilde{E}_2(k_\parallel)=(2\pi)^{-3/2}\exp\left(-\frac{k^2}{2}\right)\exp\left(-\frac{k_\parallel^2}{2}\right),\tag{11}$$

where $k=\sqrt{k_x^2+k_y^2}$. Various types of Fourier spectra were already considered (see, e.g., [1,2]) but only for the shearless stochastic magnetic field. The Eulerian autocorrelation function of the Fourier transform of the potential is

$$\langle\tilde{\psi}(\mathbf{k},k_\parallel)\tilde{\psi}(\mathbf{k}',k_\parallel')\rangle=\tilde{E}_1(k)\tilde{E}_2(k_\parallel)\delta(\mathbf{k}+\mathbf{k}')\delta(k_\parallel+k_\parallel'),\tag{12}$$

where the following definition for the Fourier transform of ψ was used:

$$\psi(x,y;z)=\int d\mathbf{k}\ dk_\parallel\tilde{\psi}(\mathbf{k},k_\parallel)\exp(i\mathbf{k}\cdot\mathbf{r}+ik_\parallel z).\tag{13}$$

The mixed Eulerian correlations between the potential and the magnetic field are defined in [5] as

$$C_{\psi n}(x,y,z)=\langle\psi(0,0,0)b_n(x,y,z)\rangle,$$

$$C_{n\psi}(x,y,z) = \langle b_n(0,0,0)\psi(x,y,z)\rangle = -\varepsilon_{nj}\frac{\partial E(x,y,z)}{\partial x_j},$$

$$n,j = x,y, \tag{14}$$

where ε_{nj} is the antisymmetric tensor ($\varepsilon 12=-\varepsilon 21=1$, $\varepsilon 11=\varepsilon 22=0$) and the following relations between these correlations hold [5]:

$$C_{\psi x}(x,y,z) = -C_{x\psi}(x,y,z) = \frac{\partial E(x,y,z)}{\partial y} = -yE(x,y,z)$$

and

$$C_{\psi y}(x,y,z) = -C_{y\psi}(x,y,z) = -\frac{\partial E(x,y,z)}{\partial x} = xE(x,y,z). \tag{15}$$

The dimensionless autocorrelation tensor for the fluctuating magnetic field components is

$$B_{ij}(x,y,z) = \langle b_i(0,0,0)b_j(x,y,z)\rangle = [\delta_{ij}(1-r^2) + x_i x_j]E(x,y,z),$$

$$i,j = x,y, \tag{16}$$

and its components are derived from $E(x,y,z)$ as [5,6]

$$B_{xx}(x,y,z) = -\frac{\partial^2 E(x,y,z)}{\partial y^2} = (1-y^2)E(x,y,z), \tag{17}$$

$$B_{yy}(x,y,z) = -\frac{\partial^2 E(x,y,z)}{\partial x^2} = (1-x^2)E(x,y,z),$$

$$B_{xy}(x,y,z) = B_{yx}(x,y,z) = \frac{\partial^2 E(x,y,z)}{\partial x\,\partial y} = xyE(x,y,z).$$

Equations (5) are the starting point of our analysis. Their solution depends on two dimensionless parameters: the magnetic Kubo number K_m and the shear parameter θ_s. The forthcoming treatment is closely following the paper of Vlad *et al.* [3].

III. DCT METHOD FOR THE MAGNETIC FIELD LINES DIFFUSION

The Langevin equations (8) will be used in order to calculate the running and consequently the asymptotic diffusion coefficient of the magnetic field lines in both small- and large-Kubo-number regimes for different values of the shear parameter. The Lagrangian correlation of the directly fluctuating parts from Eqs. (8) is defined as usual as:

$$L_{ij}(z) = K_m^2\langle b_i[\mathbf{x}(0);0]b_j[\mathbf{x}(z);z]\rangle, \tag{18}$$

where $\langle\cdots\rangle$ denotes the ensemble average over the realizations of the fluctuating magnetic field components and $\mathbf{x}=(x,y)$. The running diffusion coefficient is calculated from Eq. (18) as

$$D_{ij}(z) = \int_0^z d\zeta L_{ij}(\zeta), \tag{19}$$

provided that the stochastic field is "stationary"; the asymptotic diffusion coefficient is calculated as

$$D_{ij}^{as} = \lim_{z\to\infty} D_{ij}(z). \tag{20}$$

The main tool for determining the running and asymptotic diffusion coefficient is then the Lagrangian correlation defined in Eq. (18); an important simplification of the calculus can be done if a relation between the Lagrangian correlation and the corresponding Eulerian one can be established. Unfortunately, until now, there does not exist a general exact relation between these correlations, which is valid for both weak- and strong-turbulence regimes. However, for a weak-turbulence regime,—i.e., $K_m < 1$—an approximate formula which relates the two types of correlations exists: this is the celebrated Corrsin approximation [7,8] which includes the quasilinear and Bohm approximations. The Corrsin approximation consists in two hypothesis: (a) the statistical independence between the particle trajectories and the stochastic velocity field and (b) the displacements have a Gaussian distribution. In our paper, the geometrical point from a magnetic field line plays the role of the test particle. The Corrsin approximation is a very good approximation for a Kubo number in the range $K_m < 1$ and it can determine perturbative corrections of the diffusion coefficient (see, e.g., [6] for the shearless case). We write here, for convenience, Corrsin's relation between the Lagrangian and Eulerian correlations:

$$L_{ij}(z) = K_m^2\int d\mathbf{x}\langle b_i[\mathbf{x}(0);0]b_j[\mathbf{x};z]\delta(\mathbf{x}-\mathbf{x}(z))\rangle$$
$$\overset{Corrs}{\simeq} K_m^2\int d\mathbf{x}\langle b_i[\mathbf{x}(0);0]b_j[\mathbf{x};z]\rangle\langle\delta(\mathbf{x}-\mathbf{x}(z))\rangle. \tag{21}$$

As can be seen from Eq. (21), Corrsin assumed that, at least in some asymptotic sense, the exact propagator $\delta(\mathbf{x}-\mathbf{x}(z))$ is approximated by its ensemble average. However, at large Kubo number $K_m > 1$, the numerical simulations for already studied cases (see, e.g., [1,2,9–12]), which are similar to ours, have confirmed that the displacements are not Gaussian: a trapped particle (a geometrical point in our case) wind on almost closed paths of *small* size near the maxima or minima of the stochastic field while for small absolute values of the stochastic potential the geometrical point makes *large* displacements. This fact implies that the Corrsin approximation is not adequate for the study of a relatively strong-turbulence regime.

In our paper we use the DCT approximation, a significant step beyond the well-known Corrsin approximation. In the framework of DCT method general expressions of the running (and consequently asymptotic) diffusion coefficients can be derived for both small- and large-Kubo-number regimes. We briefly recall the main ideas of the DCT approximation; see Ref. [3]. The main idea of this method is to study the Langevin system (8) not in the whole space of realizations of the potential fluctuations; the whole space is then subdivided into *subensembles S*, characterized by given values of the potential and of the fluctuating field at the starting point of the trajectories [see below Eq. (23)]. The exact expression of the Lagrangian correlation can be written in the form of a superposition [i.e., a summing up of the contribution of each subensemble S; see below Eq. (24)] of Lagrangian correlations in the various subensembles [see below Eqs. (24) and (25)]. We mention that Eq. (24) is an exact equation. The existence of an average Eulerian velocity in the subensemble determines an average motion (decorrelation trajectory). The definition of the DCT approximation method consists practically in the following two statements.

(i) In each subensemble is defined a deterministic trajectory $\mathbf{x}^S(z)$ by the following criterion: the *Eulerian average* of the potential ψ^S in the subensemble S, calculated along this deterministic trajectory, equals the *Lagrangian* average of the same potential in the subensemble S:

$$\psi^S[\mathbf{x}^S(z);z] = \langle \psi[\mathbf{x}(z);z]\rangle^S. \tag{22}$$

This deterministic trajectory is called decorrelation trajectory.

(ii) The average Lagrangian velocity in the subensemble S is approximated with the average Eulerian velocity calculated along the deterministic trajectory [see below Eq. (29)].

Equation (22) states that in the DCT method we can consider the Lagrangian average of the potential as the corresponding Eulerian average calculated along the deterministic trajectory [i.e., the solutions of the system (27) (see below)] *in the same subensemble*. Implementing these approximations in the exact formula for the Lagrangian field correlation yields an approximation that is valid, in principle, for arbitrarily large values of K_m. The main reason for this statement is that the DCT method takes into account the trapping processes, which are neglected in previous theories based on the Corrsin approximation. The trapping process is an essential ingredient of strong turbulence theories. The validity of the approximation involved in DCT method can be assessed by *a posteriori* comparison with experiment and simulations, as is done in all theories of strong turbulence.

The DCT method is now systematically developed for the present problem. We first define a set of subensembles S of the realizations of the stochastic sheared magnetic field that are defined by given values of the potential ψ and magnetic field fluctuation $\mathbf{b}$ in the point $\mathbf{x}=0$ at the "moment" $z=0$:

$$\psi(\mathbf{0};0) = \psi^0, \quad b_i(\mathbf{0};0) = b_i^0, \quad i = x,y. \tag{23}$$

The correlation of the Lagrangian fluctuating fields defined in Eq. (5) can be represented as a sum over the subensembles S of the correlations calculated in each subensemble:

$$L_{ij}(z) = K_m^2 \int d\psi^0 d\mathbf{b}^0 P(\mathbf{b}^0,\psi^0)\langle b_i(\mathbf{0};0) b_j[\mathbf{x}(z);z]\rangle^S, \tag{24}$$

where $P(\mathbf{b}^0,\psi^0)=P(b_x^0)P(b_y^0)P(\psi^0)$, with $P(m)=(2\pi)^{-1/2}\exp(-m^2/2)$, is the probability density of $\mathbf{b}$, ψ having the values $\mathbf{b}^0,\psi^0$ at $\mathbf{x}=0$, and at the "moment" $z=0$.

Since the initial fluctuating fields in the subensemble S are $b_i(\mathbf{0};0)=b_i^0$ for all trajectories, the subensemble average defined in Eq. (24) is

$$\langle b_i(\mathbf{0};0) b_j[\mathbf{x}(z);z]\rangle^S = b_i^0 \langle b_j[\mathbf{x}(z);z]\rangle^S \tag{25}$$

and thus the Lagrangian correlation $L_{ij}(z)$ is simply the weighted average Lagrangian of the fluctuating field in all subensembles. We need first to calculate the average Eulerian fields b_i in the subensemble S.

$$b_x^S(\mathbf{x};z) \equiv \langle b_x(\mathbf{x};z)\rangle^S,$$

$$b_y^S(\mathbf{x};z) \equiv \langle b_y(\mathbf{x};z)\rangle^S. \tag{26}$$

The next step in the DCT method is to define a deterministic trajectory in each subensemble as a solution of the system (5) in which the right-hand sides are replaced by the average fields b_j^S in the subensemble. The equations for the decorrelation trajectory are thus

$$\frac{dx^S(z)}{dz} = K_m b_x^S[\mathbf{x}^S(z);z],$$

$$\frac{dy^S(z)}{dz} = K_m b_y^S[\mathbf{x}^S(z);z] + \theta_s x^S(z), \tag{27}$$

where $\mathbf{x}^S(0)=0$. In order to study the shape of the DCT we adopt the polar coordinates

$$b_x^0 = b^0 \cos\alpha, \quad b_y^0 = b^0 \sin\alpha, \tag{28}$$

where α is the angle between $\mathbf{b}^0$ and the x axis.

According to the DCT method [see the above statement (ii)], the average Lagrangian velocities in the subensemble S is approximated by the Eulerian averages calculated along the deterministic trajectories that are the solution of the system (30) written below [see Eq. (22)]:

$$\langle b_j[\mathbf{x}(z);z]\rangle^S \simeq b_j^S[\mathbf{x}^S(z);z], \quad j = x,y, \tag{29}$$

where $\mathbf{x}^S(z)$ are the solutions of the system (30); the averages from Eqs. (27) are performed using the method described in [3] and the resulting system is

$$\frac{dx^S(z)}{dz} = K_m\{-\psi^0 y^S + b^0 x^S y^S \sin\alpha + b^0[1-(y^S)^2] \times\cos\alpha\} E_1(r)E_2(z) \equiv K_m b_x^S(\mathbf{x}^S;z),$$

$$\frac{dy^S(z)}{dz} = K_m\{\psi^0 x^S + b^0 x^S y^S \cos\alpha + b^0[1-(x^S)^2]\sin\alpha\} \times E_1(r)E_2(z) + \theta_s x^S \equiv K_m b_y^S(\mathbf{x}^S;z) + \theta_s x^S. \tag{30}$$

The Eulerian correlations of the magnetic field fluctuations $b_j(\mathbf{x};z)$ in the subensemble S have been obtained as in Ref. [3] [using the usual definition of the conditional probability and Eqs. (15) and (17)]:

$$b_i^S(\mathbf{x};z) = \psi^0 C_{\psi i}(\mathbf{x};z) + b_x^0 B_{xi}(\mathbf{x};z) + b_y^0 B_{yi}(\mathbf{x};z), \quad i = x,y. \tag{31}$$

In deriving Eq. (30) we have made the following usual approximation, specific to the DCT method [5]: *we have considered that the contribution of the subensemble-averaged shear term* $\langle \theta_s x(z)\rangle^S$ *in Eqs.* (26), *in each subensemble* S, *is equal to its value along the deterministic trajectory*, $\theta_s x^S(z)$.

The Lagrangian average of $x(z)$ in the subensemble S is then approximated by x^S.

Using Eqs. (25) and (29), for arbitrary values of the dimensionless parameters K_m, θ_s, and α and given Eulerian correlations [see Eqs. (14)–(17)] of the fluctuating magnetic field components that exist in Eq. (5), the Lagrangian correlation tensor given in Eq. (24) has the following components:

$$L_{xx}(z) = (2\pi)^{-3/2} K_m^2 \int_0^{2\pi} d\alpha \int_{-\infty}^{\infty} d\psi^0 \int_0^{\infty} db^0 (b^0)^2 \cos\alpha \times \exp\left[-\frac{(\psi^0)^2 + (b^0)^2}{2}\right] b_x^S[\mathbf{x}^S(z);z], \tag{32}$$

$$L_{xy}(z) = (2\pi)^{-3/2} K_m^2 \int_0^{2\pi} d\alpha \int_{-\infty}^{\infty} d\psi^0 \int_0^{\infty} db^0 (b^0)^2 \cos\alpha \times \exp\left[-\frac{(\psi^0)^2 + (b^0)^2}{2}\right] b_y^S[\mathbf{x}^S(z);z],$$

$$L_{yx}(z) = (2\pi)^{-3/2} K_m^2 \int_0^{2\pi} d\alpha \int_{-\infty}^{\infty} d\psi^0 \int_0^{\infty} db^0 (b^0)^2 \sin\alpha \times \exp\left[-\frac{(\psi^0)^2 + (b^0)^2}{2}\right] b_x^S[\mathbf{x}^S(z);z],$$

$$L_{yy}(z) = (2\pi)^{-3/2} K_m^2 \int_0^{2\pi} d\alpha \int_{-\infty}^{\infty} d\psi^0 \int_0^{\infty} db^0 (b^0)^2 \sin\alpha \times \exp\left[-\frac{(\psi^0)^2 + (b^0)^2}{2}\right] b_y^S[\mathbf{x}^S(z);z],$$

where $b_x^S[\mathbf{x}^S(z);z]$ and $b_y^S[\mathbf{x}^S(z);z]$ are identified from Eq. (31) and are calculated with the solutions of the system (30). Integrating Eq. (32) with respect to z we obtain the running diffusion tensor components D_{ij}.

The analysis of the diffusion tensor components will be the object of Sec. V of the paper.

IV. DECORRELATION TRAJECTORIES

We consider now the solutions that are obtained by a numerical integration of the system (30). A specified trajectory depends on the parameters that define the subensemble S: ψ^0, b^0, and α. It depends also on the magnetic Kubo number K_m and on the shear parameter θ_s. In the following example we choose a subensemble S defined by the following values of the parameters: $\psi^0=2$, $b^0=1$, and $\alpha=\pi/3$. We also choose three different values of the Kubo number K_m, $K_m=0.1$, $K_m=1$, $K_m=3$ and for each of them three different values for the shear parameters θ_s, $\theta_s=0$, $\theta_s=1$, and $\theta_s=6$. In Figs. 1–3, the DCT are shown; for all pictures the dotted curves correspond to $\theta_s=0$, the dashed ones to $\theta_s=1$, and the solid ones to $\theta_s=6$.

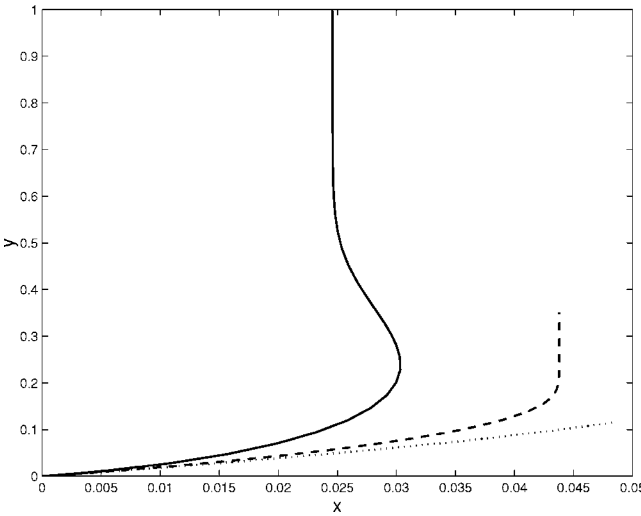

FIG. 1. Decorrelation trajectories for $K_m=0.1$ Solid line: $\theta_s=6$. Dashed line $\theta_s=1$. Dotted line $\theta_s=0$.

The general shape of the decorrelation trajectories consists of a first, more or less localized portion, followed by a linear portion parallel to the y axis. The latter appears when the Lagrangian field correlation—or, equivalently, the Lagrangian average field—has been damped out by the factor $E_2(z)=\exp(-z^2/2)$. For large values of z, $x^S(z)\approx x^S$ and $y^S(z)\approx \theta_s x^S z$. For small K_m as in Fig. 1 ($K_m=0.1$) the localized portion is short (weak trapping) and is essentially produced by the effect of the shear (e.g., $\theta_s=6$).

In Fig. 2 ($K_m=1$), the trajectory for large shear ($\theta_s=6$) exhibits strong trapping, performing a turn before escaping. For larger nonlinearity ($K_m=3$, Fig. 3) all trajectories are strongly trapped: for $\theta_s=6$ the DCT performs two turns before escaping. The competition between the Kubo number and the shear parameter will be more clearly exhibited in the

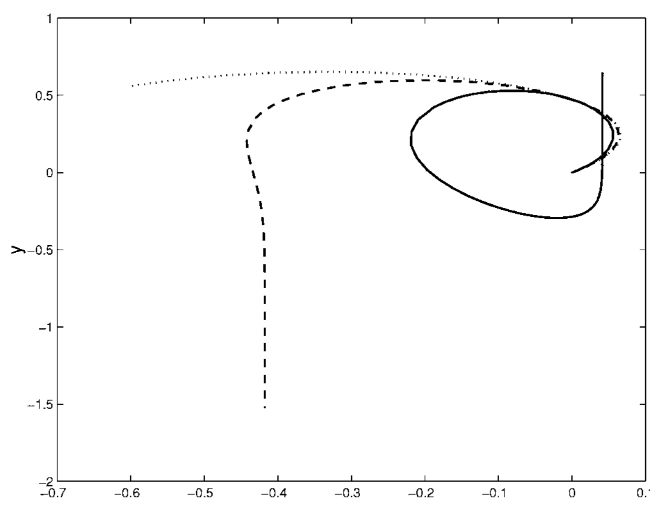

FIG. 2. Same as in Fig. 1 but for $K_m=1$.

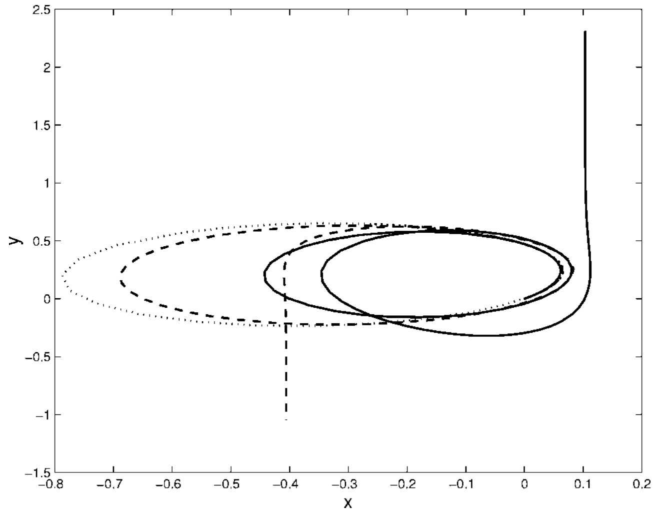

FIG. 3. Same as in Figs. 1 and 2 but for $K_m=3$.

next section when the diffusion coefficient behavior will be examined.

V. DIFFUSION COEFFICIENTS

A computer code based on the Runge-Kutta-Fehlberg 45 (RKF45) method has been developed [13]. Using this code we have calculated the Lagrangian correlation tensor and the running and asymptotic diffusion tensor components. It determines the decorrelation trajectories [Eq. (30)] for a large enough number of subensembles and performs the integrals in Eqs. (32) and (19). A careful analysis of the integrands permits the optimization of the choice of the parameters. For the evaluation of the Lagrangian correlation tensor we have calculated $120\times 31\times 31$ decorrelation trajectories; for any decorrelation trajectory we have used 200 points for "time" and the final integration time (z final) was 10—i.e., a safe value in order to reach the asymptotic regime (see Figs. 4–6). For an arbitrary value of α the diffusion tensor is non-diagonal; the shear term breaks the isotropy of space and introduces the dependence on the angle α. This largely increases the computation time.

In Figs. 4–6 the running diffusion tensor components are represented for three different values of the Kubo number K_m: $K_m=0.1$, $K_m=1$, and $K_m=3$. For each Kubo number four different values of the shear parameter have been chosen: $\theta_s=0$ (dotted curves), $\theta_s=0.2$ (dash-dotted curves), $\theta_s=1$ (dashed curves), and $\theta_s=6$ (solid curves) in all pictures.

In Fig. 4(a), the influence of the shear is exhibited in a decrease of the asymptotic value $D_{xx}^{as}$of the component $D_{xx}(z)$; for $z\geqslant 3$ the asymptotic regime is practically achieved for all values of the shear parameter. In Fig. 4(d), the asymptotic values D_{yy}^{as} of $D_{yy}(z)$ are almost the same for allvalues of the shear parameter; comparing the asymptotic values for $\theta_s=0.2$ from Figs. 4(a) and 4(d) we can state that $D_{xx}^{as}\approx D_{yy}^{as}$. In Fig. 4(c) there is an obvious increase in absolute value of the asymptotic value $|D_{yx}^{as}|$ when the shear parameter increases; the same behavior is manifest for D_{xy}^{as} as we can see from Fig. 4(b). In the limit of a reasonable error ($\approx 10^{-3}$) the asymptotic value $|D_{yx}^{as}|$ for $\theta_s=0.2$ is practically

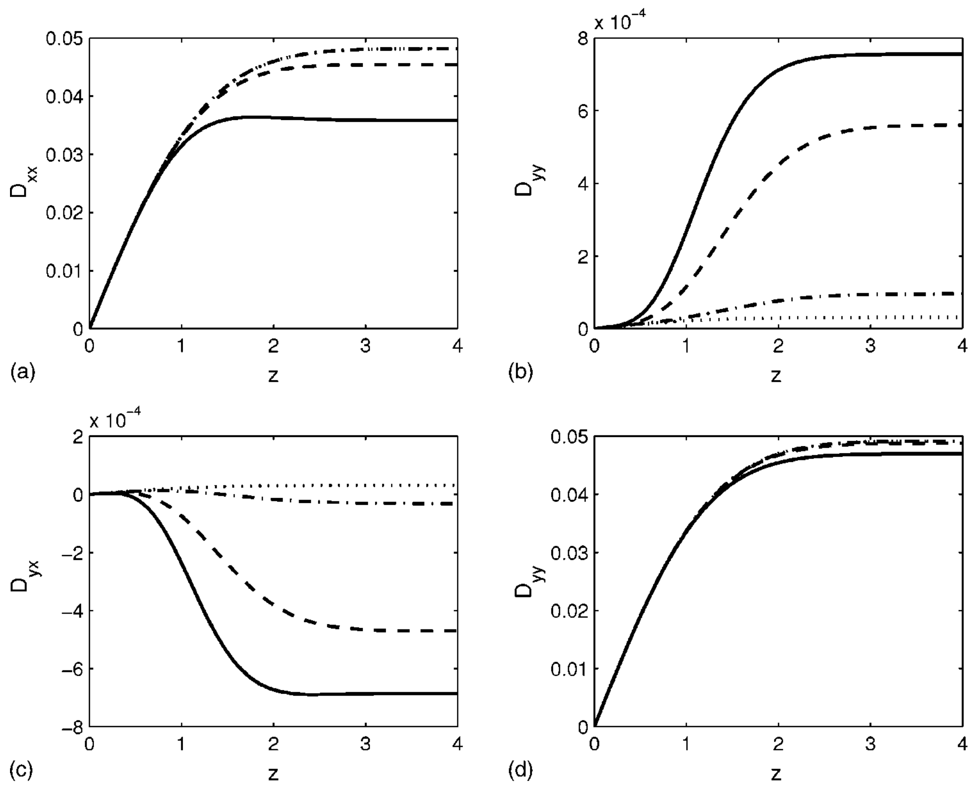

FIG. 4. The running diffusion coefficients for $K_m=0.1$ and for four values of the shear parameter. In all pictures $\theta_s=0$ (dotted line), $\theta_s=0.2$ (dash-dotted line), $\theta_s=1$ (dashed line), and $\theta_s=6$ (solid line).

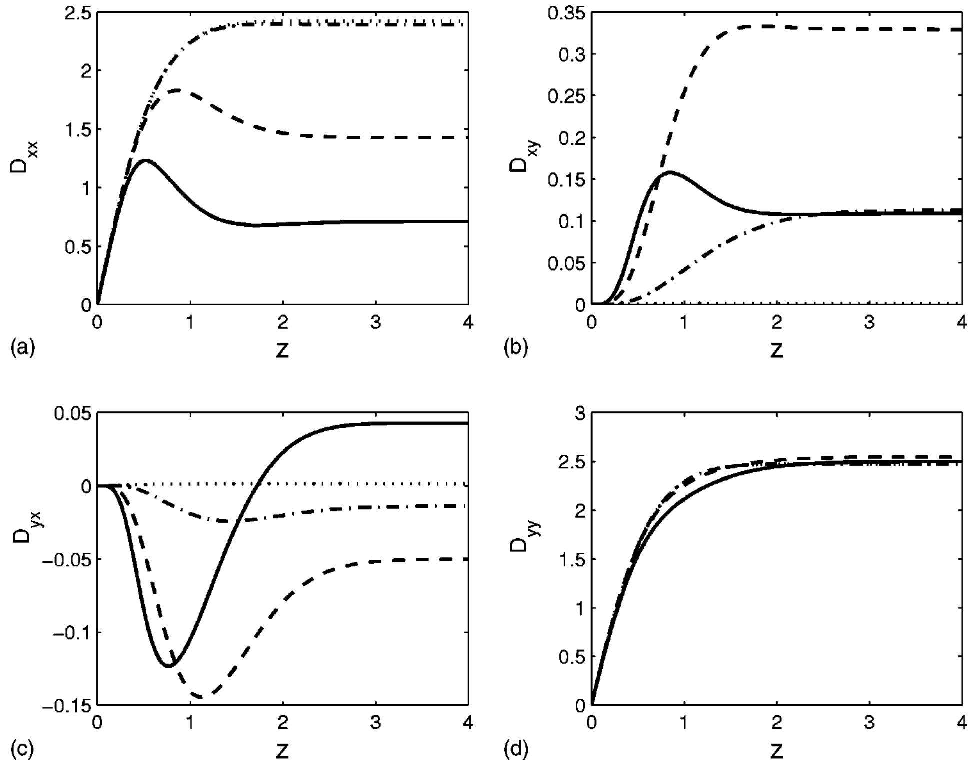

FIG. 5. Same as in Fig. 4 but for $K_m=1$.

zero. At the same time, also for $\theta_s=0.2$, $D_{xy}^{as}/D_{xx}^{as} \approx D_{xy}^{as}/D_{yy}^{as}<1$ These results are in agreement with those obtained in a previous work [4]. In all pictures from Fig. 4 all components $D_{jn}(z)$ of the diffusion tensor exhibit a monotonous increase (in absolute value) followed by a saturation. This means that there is no trapping in this case for any value of $\theta_s<6$.

In the case of intermediate Kubo number ($K_m=1$, Fig. 5) a number of new features appear. The component $D_{xx}(z)$ is strongly affected by the shear. When $\theta_s \geqslant 1$ the correspond-

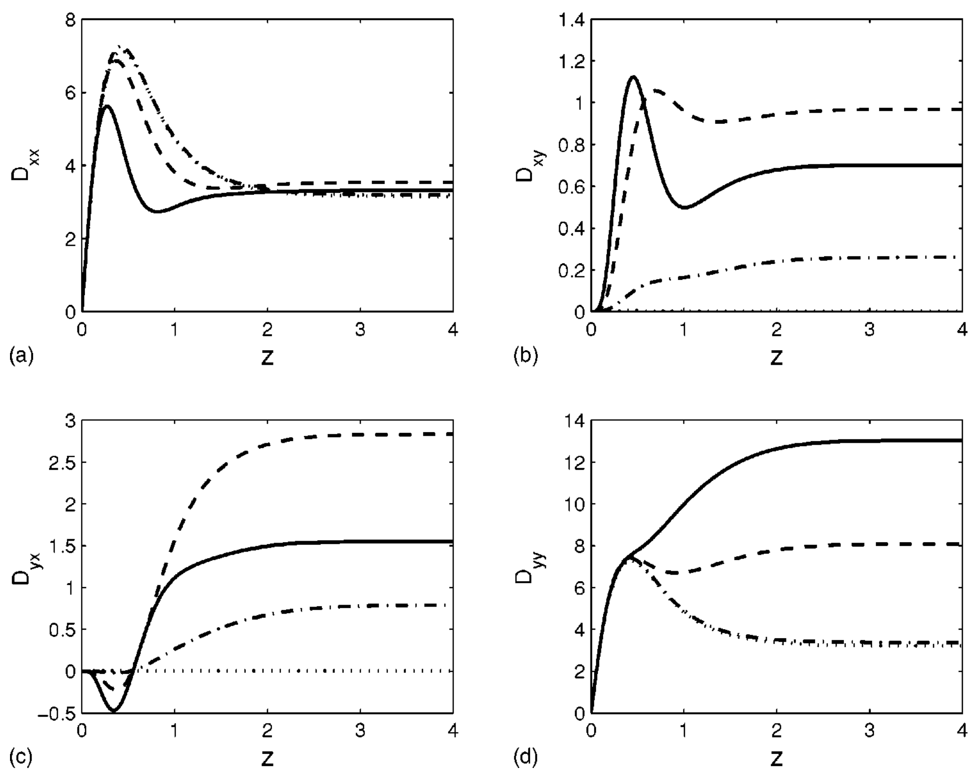

FIG. 6. Same as in Figs. 4 and 5 but for $K_m=3$.

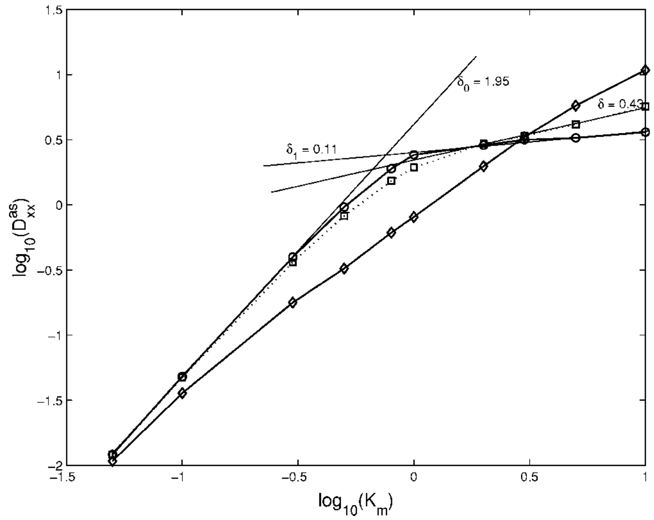

FIG. 7. The diagonal asymptotic diffusion coefficient D_{xx}^{as} as a function of the magnetic Kubo number K_m for different values of the shear parameter in a $\log_{10}$-$\log_{10}$ plot. Diamonds: $\theta_s=6$. Squares: $\theta_s=1$. Circles: $\theta_s=0$.

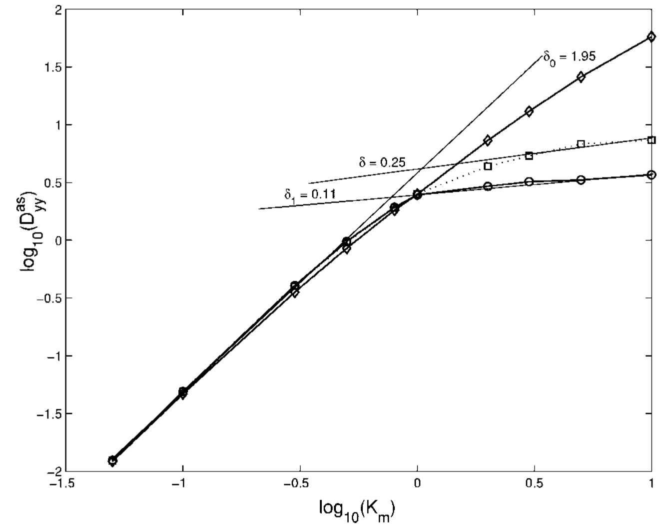

FIG. 8. Same as in Fig. 4 but for D_{yy}^{as}.

ing function presents a maximum and the asymptotic value is significantly smaller than for $\theta_s=0$. This is a clear signature of the trapping effect. On the other hand, the coefficient $D_{yy}(z)$ is almost unaffected by the presence of the shear. The nondiagonal coefficients are strongly affected by the shear: their behavior is quite different from the small-K_m regime. The coefficient $D_{yx}(z)$ exhibits a minimum [at the same value of z as the maximum ofthe corresponding $D_{xx}(z)$], followed by a saturation at a value which becomes positive for large θ_s. The component $D_{xy}(z)$ has a behavior qualitatively similar to $D_{xy}(z)$, but the trapping effect appears for higher values of θ_s.

In the case of large K_m ($K_m=3$, Fig. 6) all these features are enhanced. For the diagonal component $D_{xx}(z)$ ("radial" running diffusion coefficient) the trapping effect is present even for $\theta_s=0$, and is accompanied (for $\theta_s=6$) by the appearance of a minimum before the saturation. Unexpectedly, for $D_{yy}(z)$ the shear has an inverse effect. The trapping is strong for small θ_s (marked peak), becomes weaker for $\theta_s=1$, and is barely present for $\theta_s=6$. Strong shear produces a marked negative minimum in $D_{yx}(z)$, followed by asaturation at a positive value. The behavior of $D_{xy}(z)$ is qualitatively similar to $D_{xx}(z)$.

We now discuss the dependence of the asymptotic diffusion coefficients on K_m and on θ_s, limiting ourselves to the diagonal ones.

In Figs. 7 and 8 the coefficients D_{xx}^{as}, D_{yy}^{as} are plotted versus K_m (in a $\log_{10}-\log_{10}$ representation) for three values of $\theta_s=0,1,6$. We first note the common feature: for small values of K_m, in almost all cases, the curves start with a slope $\delta_0 \simeq 1.95$, very close to the expected quasilinear value $\delta=2$ [4]. There is no significant trapping in this region, except for D_{xx}^{as} for $\theta_s=6$, in which case the curve departs earlier from the quasilinear regime.

For $K_m>1$ the trapping becomes important and produces a final slope smaller than 2, hence an important deviation from the quasilinear value. We stress the fact that in absence of shear the slope of both coefficients D_{xx}^{as} and D_{yy}^{as} (which are equal), $\delta_1 \simeq 0.11$, is much smaller than the one appearing in the equivalent problem treated in [3]. In the latter case the Eulerian potential correlation was assumed to be (in the present notation) $E_1(r) \sim [1+r^2/2n]^{-n}$, $0.5<n<2$, and $E_2(z) \sim \exp(-z)$, in contrast to the present choice, Eq. (10). Thus, the Gaussian correlation produces much stronger trapping than the Lorentzian. This fact was noticed for the electrostatic guiding center diffusion.

For both diagonal coefficients, the presence of shear produces a final slope larger than the shearless one, but still smaller than the quasilinear value. For $\theta_s=1$ this slope is $\delta \simeq 0.43$ for D_{xx}^{as} and $\delta \simeq 0.25$ for D_{yy}^{as}. For $\theta_s=6$, the final slope is not yet reached in Figs. 7 and 8. Its determination would require higher values of K_m, for which the numerical calculations become very strongly time consuming.

VI. CONCLUSIONS

In most previous works the problem of the diffusion of magnetic lines is treated by starting either from Langevin equations (as in the present paper) or from a hybrid kinetic (or stochastic Liouville) equation and applying a strong approximation. In [4] the hybrid kinetic equation is treated within the quasilinear approximation, thus yielding the scaling $D_{xx}^{as}(K_m) \approx K_m^2$. In [6] the Lagrangian magnetic field correlation is evaluated by using the well-known Corrsin approximation [7]. The latter yields the quasilinear result for small K_m and the Bohm scaling $D_{xx}^{as}(K_m) \approx K_m$ for large K_m. The latter scaling is known, however, to be incorrect. Indeed, the Corrsin approximation ignores the trapping effect which necessarily exists in a strongly turbulent plasma [14,15].

The method of the decorrelation trajectories was specifically designed in order to take the latter effect into account [3]. It was applied in previous works to various plasma turbulence situations or, in particular, to the diffusion of guiding centers in presence of a fluctuating electrostatic potential and a constant magnetic field [3]. The latter problem is equivalent to the present one for $\theta_s = 0$.

In the present paper we analyzed the influence of a sheared reference magnetic field on the diffusion of fluctuating magnetic lines. We applied to this problem the decorrelation trajectory method.

In Sec. IV we exhibited a selection of deterministic *decorrelation trajectories* for various values of K_m and of θ_s. Increasing values of the latter parameter produce an incomplete oscillation around the starting point before the final linear escape in the y direction. This *individual trapping* effect is more pronounced, the larger K_m and larger θ_s.

In Sec. V we examined the global combined effect of K_m and of θ_s. The shape of the *running diffusion coefficients* provides an interesting insight into the transient behavior of these quantities (Figs. 4–6). The diagonal coefficients start with a linear part, defining a *ballistic regime* $D_{jj}(z) \sim z$ but the nondiagonal coefficients have adifferent start: $D_{jn}(z) \sim z^p (p > 1, j \neq n)$. In all of them a *trapping effect* occurs for large enough K_m and/or θ_s. This effect consists of a temporary inversion of the monotonous growth toward the asymptotic value, with the appearance of a maximum at z_M or even an oscillation, as in Fig. 6. This trapping regime does not appear simultaneosly and not with the same strength in the four running diffusion coefficients, thus providing various shapes shown in Figs. 4–6.

Finally the behavior of the *asymptotic diffusion coefficients* is shown in Fig. 7. The somewhat unexpected feature appearing here is that the shear *increases* the final slope of the $\log_{10} D_{jj}^{as}$ vs $\log_{10} K_m$ curves. Thus the *global trapping effect* is weaker for larger θ_s, whereas the *transient trapping effect* [e.g., in $D_{xx}(z)$] is enhanced by the shear.

In all cases (except $\theta_s = 6$ for which the final slope was not reached) the final slope is below the Bohm value $\delta = 1$.

ACKNOWLEDGMENTS

Two of us (M.N. and I.P.) would like to acknowledge the warm hospitality of the members of Laboratoire de Physique Statistique et Plasmas, Universite Libre de Bruxelles, Belgium. We also want to thank Dr. M. Vlad and Dr. J. H. Misguich for very fruitful discussions. This work was supported by Association EURATOM-MEC, Romania.

[1] P. Castiglione, J. Phys. A **33**, 1975 (2000).
[2] M. Ottaviani, Europhys. Lett. **64**, 4 (1992).
[3] M. Vlad, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **58**, 7359 (1998).
[4] E. Vanden Eijnden and R. Balescu, Phys. Plasmas **3**, 815 (1996).
[5] M. Vlad, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **63**, 066304 (2001).
[6] Hai-Da Wang, M. Vlad, E. Vanden Eijnden, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **51**, 4844 (1995).
[7] W. D. Mc Comb, *The Physics of Fluid Turbulence* (Clarendon, Oxford, 1990).
[8] S. Corrsin, in *Atmospheric Diffusion and Air Pollution*, edited by F. N. Frenkiel and P. A. Sheppard (Academic, New York, 1959).
[9] M. Vlad, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **66**, 038302 (2002).
[10] J. H. Misguich, J.-D. Reuss, M. Vlad, and F. Spineanu, Phys. Mag. **20**, 103 (1998).
[11] J.-D. D. Reuss and J. H. Misguich, Phys. Rev. E **54**, 1857 (1996).
[12] J.-D. Reuss, M. Vlad, and J. H. Misguich, Phys. Lett. A **241**, 94 (1998).
[13] G. E. Forsythe, M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations* (Prentice-Hall, Englewood Cliffs, NJ, 1977).
[14] R. H. Kraichnan, Phys. Fluids **13**, 22 (1970).
[15] J. A. Crottinger and T. H. Dupree, Phys. Fluids B **4**, 2854 (1992).